

Instructions- (i) All questions are compulsory. (ii) Read instructions carefully of the question paper and then answers of the questions. (iii) Question paper has two sections - Section - ' A ' and Section - ' B '. (iv) In the Section - ' A ' Question Nos. 1 to 5 are objective type, which contain the - choose the correct option, answer in one word/sentence, fill in the blanks, True/False and match the columns. Each question carries 5 marks. (v) In the Section - ' B ' question Nos. 6 to 24 has Internal option. (vi) Q.Nos. 6 to 10 carry 2 marks each. (vii) Q.Nos. 11 to 17 carry 4 marks each. (viii) Q.Nos. 18 to 22 carry 5 marks each. (ix) Q.Nos. 23 and 24 carry 6 marks each.

Section ' A '

Q.1. Choose the correct options- (5 × 1 = 5)

(a) Fraction form of $\frac{1}{(x+3)(x+4)}$ is-

(i) $\frac{1}{(x+3)} + \frac{1}{(x+4)}$

(ii) $\frac{1}{(x+3)} - \frac{1}{(x+4)}$

(iii) $\frac{1}{(x+4)} - \frac{1}{(x+3)}$

(iv) $\frac{1}{2} \left[\frac{1}{x+3} + \frac{1}{x+4} \right]$

(b) The perpendicular distance of the plane $3x - 6y + 5z = 12$ from origin is be-

(i) $\frac{-\sqrt{70}}{12}$

(ii) $\frac{-12}{\sqrt{70}}$

(iii) $\frac{12}{\sqrt{70}}$

(iv) $\frac{\sqrt{70}}{12}$

(c) The unit vector in the direction of " $\hat{i} + \hat{j} + \hat{k}$ " is be-

(i) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

(ii) $\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$

(iii) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j} + \hat{k})$

(iv) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$

(d) Differential coefficient of " $\log(\sin x)$ " with respect to 'x' is-

(i) $\cot x$

(ii) $\operatorname{cosec} x$

(iii) $\tan x$

(iv) $\sec x$

(e) By Newton-Raphson's method the formula for finding the square root of any number "y" is-

(i) $x_{n+1} = \frac{1}{2} \left[x_n + \frac{y}{x_n} \right]$

(ii) $x_{n+1} = \frac{1}{2} \left[x_0 + \frac{y}{x_0} \right]$

(iii) $x_{n+1} = \frac{1}{3} \left[2x_n + \frac{y}{x_n^2} \right]$

(iv) $x_{n+1} = \frac{1}{3} \left[2x_0 + \frac{y}{x_0^2} \right]$

Q.2. Answers in one word/sentences-

(5 × 1 = 5)

(i) Write the equation of a straight line which passes through the point (2, 1, 3) and has direction-ratios (1, 3, 2)

(ii) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of the triangle ABC, then write the formula of the area of ΔABC .

(iii) Write the value of $\int \frac{dx}{ax+1}$.

(iv) Define the positive co-relation.

(v) What is the value of $\sqrt{12}$ by Newton-Raphson's method after first iteration?

Q.3. Fill in the blanks- (5 × 1 = 5)

- (i) Is $\sin^{-1} x + \cos^{-1} x = \dots\dots\dots$
- (ii) Sphere $3x^2 + 3y^2 + 3z^2 - 6x - 12y + 6z + 2 = 0$ has centre $\dots\dots\dots$
- (iii) If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$ will be $\dots\dots\dots$
- (iv) The arithmetic mean of regression coefficients always $\dots\dots\dots$ the correlation.
- (v) Related to the numerical method, the formula by the trapezoidal rule is $\dots\dots\dots$

Q.4. Write the True/False- (5 × 1 = 5)

- (i) Distance the point P (x, y, z) from the plane- X-Y is $\sqrt{x^2 + y^2 + z^2}$.
- (ii) Differential coefficient of e^x with respect to $\sqrt{x}$ is $\sqrt{x} \cdot e^x$.
- (iii) $f(x) = 2x^3 - 21x^2 + 36x - 30$ is maximum at $x = 1$.
- (iv) According to the Newton-Raphson's method the approximate root of the equation $f(x) = 0$ is x_n then be $x_n = x_{n+1} - \frac{f(x_n)}{f'(x_n)}$.
- (v) By the method of Newton-Raphson, the cube root of 10, after first iteration is 2.167.

Q.5. Match the correct pair- (5 × 1 = 5)

'A'

'B'

(a) $\int \frac{dx}{x^2 + a^2}$

(i) $\log \left[x - \sqrt{x^2 - a^2} \right]$

(b) $\int \frac{dx}{\sqrt{a^2 - x^2}}$

(ii) $\frac{1}{a} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{x}{a}$

(c) $\int \sqrt{a^2 - x^2} dx$

(iii) $\frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$

(d) $\int \frac{dx}{\sqrt{x^2 - a^2}}$

(iv) $a \cdot \tan^{-1} x$

(e) $\int \sqrt{a^2 + x^2} dx$

(v) $\sin^{-1} \left(\frac{x}{a} \right)$

$$(vi) \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log \left[x + \sqrt{x^2 + a^2} \right]$$

$$(vii) \log \left[x + \sqrt{x^2 - a^2} \right]$$

Section 'B'

Q.6. Prove that-

2

$$\vec{AB} + \vec{BC} + \vec{CA} = 0$$

(Or) If $\vec{OP} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{OQ} = 2\hat{i} - 2\hat{j} - \hat{k}$ then find the modulus of $\vec{PQ}$.

Q.7. Prove that vectors $2\hat{i} - 3\hat{j} + 5\hat{k}$ and $-2\hat{i} + 2\hat{j} + 2\hat{k}$ are mutually perpendicular.

(Or) If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j}$, then find $\vec{a} \times \vec{b}$.

Q.8. Find the vector equation of sphere whose centre is $(2, -3, 4)$ and radius is 5.

2

(Or) Find the distance of point $(2, -1, 3)$ from the plane

$$\vec{r} \cdot (3\hat{i} + 2\hat{j} - 6\hat{k}) + 15 = 0.$$

Q.9. Evaluate- $\int \frac{dx}{1 + \cos 2x}$

2

(Or) Evaluate- $\int \frac{1}{1 - 4x} dx$

Q.10. Evaluate- $\int_0^{\pi/4} \sin 2x dx$.

2

(Or) Evaluate- $\int \frac{\sec x}{(\sec x - \tan x)} dx$.

Q.11. Resolve the following fraction into partial fractions- $\frac{16}{(x+2)(x^2-4)}$

(Or) Resolve the following fraction into partial fractions- $\frac{2x+1}{(x-1)(x^2+1)}$

Q.12. Prove that-

4

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1} \left[x\sqrt{1-y^2} + y\sqrt{1-x^2} \right].$$

(Or) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then prove that, $xy + yz + zx = 1$.

Q.13. Find the differential coefficient of $\sin x$ by first principle. 4

(Or) If $y = \log (\log \sin x)$, then evaluate $\frac{dy}{dx}$.

Q.14. Differentiate, $\tan^{-1} \left[\frac{\cos x + \sin x}{\cos x - \sin x} \right]$ with respect to x . 4

(Or) Find the differential coefficient with respect to x of $\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$.

Q.15. If the edge of a cube is increasing at the rate of 5cm/sec., find the rate of increasing of its volume when its edge is 8cm long? 4

(Or) Prove that, $f(x) = x^3 - 3x^2 + 3x - 100$ is an increasing function in R .

Q.16. If "r" is a coefficient of correlation of two variables x and y , then prove that-

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x \cdot \sigma_y} \text{ Where } \sigma_x^2, \sigma_y^2 \text{ and } \sigma_{x-y}^2 \text{ are the variance of } x, y$$

and $x - y$ respectively. 4

(Or) If $n = 10$, $\sum x = 50$, $\sum y = 30$, $\sum x^2 = 290$, $\sum y^2 = 300$, $\sum xy = -115$, then find the coefficient of correlation.

Q.17. If " θ " be the angle between two regression lines and regression coefficients are $b_{yx} = 1.6$ and $b_{xy} = 4$, then find the value of $\tan \theta$. 4

(Or) Prove that coefficient of correlation is the Geometric mean of regression co-efficients.

Q.18. If $\cos \alpha$, $\cos \beta$, $\cos \gamma$ are direction cosines of any straight line, then prove that- $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ 5

(Or) Equation of the sphere is, $2x^2 + 2y^2 + 2z^2 - 8x + 12y - 16z + 8 = 0$ find its centre and radius.

Q.19. Prove that- $\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) = 1$ 5

(Or) Evaluate- $\lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x} \right)$.

Q.20. Find the area of circle, $x^2 + y^2 = a^2$. 5

(Or) Prove that- $\int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \frac{1}{2} \log 2$.

Q.21. Solve the Differential Equation, 5

$$(1+x)y \, dx + (1-y)x \, dy = 0.$$

(Or) Solve the differential equation, $(x^2 + xy) \, dy = (x^2 + y^2) \, dx$.

Q.22. Write theorem of total probability and prove it. 5

(Or) A bag contains 8 black and 5 white balls. 2 balls are drawn. Find the probability that both the balls are white.

Q.23. Prove that the lines. 6

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Find also the point of intersection.

(Or) Find the equation of the sphere which passes through the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ and whose centre lies on the plane $3x - y + z = 2$.

Q.24. Prove by vector method. 6

$$\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B.$$

(Or) Find the shortest distance between the lines.

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + t(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ and } \vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + s(3\hat{i} + 4\hat{j} + 5\hat{k}).$$

